Mathematics with Maple

A Comprehensive E-Book

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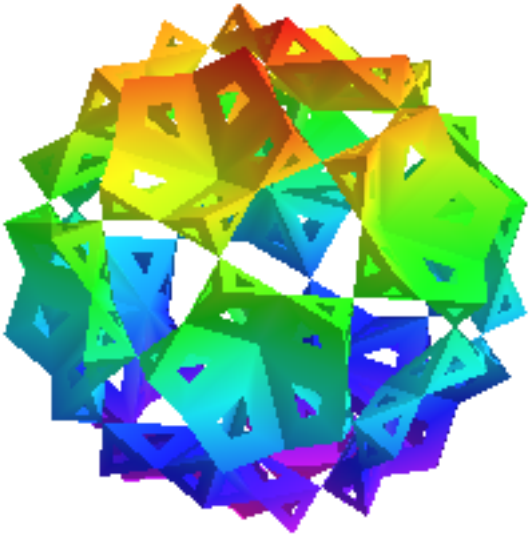

Preface

The main objective of these Maple worksheets, organized for use with all Maple versions from Maple 14, is to show how the computer algebra system Maple can be integrated as a valuable and powerful tool into the learning process of the material found in most mathematics textbooks at university levels. This interactive e-book can be used as a supplement to a standard textbook or on its own. All chapters in the worksheets summarize mathematical concepts and principles associated with each topic.

There are more than 600 worked examples, presented in a dynamic and interactive form. These show how Maple can be used to solve problems and also reduce the amount of time spent in routine algebraic manipulation. With Maple there is a great opportunity for exploration, clarify concepts, trying different functions, zoom in on an area, visualize mathematical objects in two or

three dimensions, perform animations etc.

I do assume that you have a basic familiarity with Maple, that you can start Maple, know how to enter commands, etc. If not, I recommend you to go through the Getting Started Guide This manual describes how to use the Standard Maple interface, which have two modes, document mode and worksheet mode. All worksheets in **Mathematics with Maple** are in worksheet mode, and all entering inputs are in text mode (maple input). Right-click the mouse and select **Convert To 2-D Math** if you want an input as you will find in mathematical textbooks. If you need an overview of all Maple features and detailed help related to performing computations, creating plots, interactive documents etc., please consult the User Manual. You can also go to The Maple Portal.

My own experience is that Maple's closeness to mathematical thinking, shortens the learning time of the syntax significantly. The body of text presents only the formal mathematics concepts needed for the worked solutions of the examples, which are familiar problems found in most mathematical textbooks, so that both the commands and the style of interaction with Maple can be mastered by students and others in a relatively short period of time.

To get the most from these worksheets, you should work through the examples as they occur and in conjunction with the textbook you are using. Be aware that Maple can be used in several ways:

- Only as a solving engine. That means, you enter a mathematical expression which Maple subsequently execute, and you are freed from tedious computations, especially if you are not interested in how Maple gets the answer to a given command
- To present a method as a sequence of steps in much the same way as a textbook would, and illustrate the method with simple examples. The primary purpose here is to present intermediate steps to show you how to solve the problem. This is probably relevant in most of a student's homework solutions
- Unlike a math textbook you are free to execute larger and more realistic examples, and you can explore interactively how a solution of a problem depends on various parameters of the problem under study. The graphics capabilities are extremely helpful for visualizing the behavior of the system under investigation

In the examples in the worksheets, I have tried to illustrate these methods of using Maple. The list of contents and all commands are hyperlinked to provide easy access to all chapters, and to the help pages for the commands used in the worksheets. Then you can assure that you are using the commands correctly, see Maple's own examples on the help pages, and get further help for the commands, Maple features, packages and more. You will also find hyperlinks to famous mathematicians on the web.

A typical working session will begin by opening the contents section below. Select the link to the chapter and section you want to study. Open and press the **Enter** key repeatedly to execute in sequence the Maple commands that are already provided, or press the key to execute the entire worksheet.

Hopefully these samples worksheets will serve as a guide when you prepare your own worksheets solutions to exercises and problems in math or other homework where Maple is used, and also inspire you to concentrate on exploring the many facets of mathematics. But, please bear in mind: You must do the thinking, Maple alone only does the calculations and move you more quickly into applications of problem solving.

One of the beautiful qualities of Maple is that much can be done with few commands. Most of the examples in these worksheets involve no formal programming with Maple. I have included two

extra library packages, the calc package and the calcplot package. These are not part of the Maple software. The procedures in these packages are designed for visualizing concepts from calculus and advanced calculus, without any need for you to learn the subtleties of Maple's base commands in order to use the procedures. Many of the visualization procedures go far beyond Maple's built in visualization tools.

My hope is that these worksheets will give you as a student easier access and additional help to make effective use of Maple, in classes, labs, homework and projects, and in future jobs. Maple allows the level of mathematical typesetting and also necessary word processing facilities needed for writing sets of exercises, homework and professional reports.

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This Maple project is financially supported by *Norwegian Non-Fiction Writers and Translators Association.*

Contents

1 Functions and Graphs
1.1 Functions and Inverse Functions
1.2 Graphs of Equations and Functions
1.3 Polynomials and Algebraic Functions
1.4 Exponential and Logarithmic Functions
1.5 Trigonometric Functions and Their Inverses
1.6 Hyperbolic Functions and Their Inverses

2 Limits and Continuity
2.1 The Limit Concept
2.2 Theorems Involving Limits
2.3 Continuous Functions

3 Derivatives and Differentials
3.1 Average Rate of Change of a Function
3.2 Derivative of a Function
3.3 Differentiation Rules
3.4 The Chain Rule

4 Integration
4.1 Indefinite Integral
4.2 Area, Riemann Sums, and the Definite Integral
4.3 Integration Methods
4.4 Improper Integrals

3.5 Implicit Differentiation
3.6 Extreme Values and Optimization
3.7 Rolle's Theorem and the Mean Value Theorem
3.8 Indeterminate Forms

5 Applications of Integrals
5.1 Area Between Two Curves
5.2 Volumes
5.3 Arch Length and Surface Area of Revolution
5.4 Mass, Moments and Centroids
5.5 Physical Applications

6 Vectors, Complex Numbers, and Linear Algebra
6.1 Vectors
6.2 Complex Numbers
6.3 Systems of Linear Equations
6.4 Matrix Algebra
6.5 Determinants
6.6 Vector Spaces
6.7 Eigenvalues and Eigenvectors
6.8 Linear Transformations
6.9 Orthogonality and Least Squares
6.10 Symmetric Matrices and Quadratic Forms
6.11 Applications of Linear Algebra

▼7 Ordinary Differential Equations
7.1 Differential Equations and Initial Value Problems
7.2 First Order Equations
7.3 Second Order Linear Equations
7.4 Higher Order Linear Equations
7.5 Applications

8 Conics, Polar Coordinates, and Parametric Curves
8.1 Conic Sections
8.2 Polar Coordinates
8.3 Parametric Curves

▼9 Curves and Surfaces in Space

10 Multivariable Functions and Partial

9.1 Lines and Planes in Space
9.2 Vector-Valued Functions and Space Curves
9.3 Curvature and Acceleration
9.4 Cylinders and Quadratic Surfaces
9.5 Cylindrical and Spherical Coordinates

Derivatives
10.1 Functions of Several Variables
10.2 Limits and Continuity
10.3 Partial Derivatives
10.4 The Chain Rule
10.5 Directional Derivatives, The Gradient, Tangent Plane and Normal Line
10.6 Multivariable Optimization

11 Multiple Integration
11.1 Double Integrals
11.2 Iterated Double Integrals
11.3 Double Integrals in Polar Coordinates
11.4 Surface Area
11.5 Triple Integrals
11.6 Integration in Cylindrical and Spherical Coordinates
11.7 Change of Variables in Multiple Integrals

12 Vector Analysis
12.1 Vector Fields
12.2 Line Integrals
12.3 Independence of Path and Conservative Vector Fields
12.4 Green's Theorem
12.5 Surface Integrals
12.6 Stokes' Theorem
12.7 The Divergence Theorem

13 Systems of Differential Equations
13.1 First Order Systems
13.2 Homogeneous Linear Systems
13.3 Nonhomogeneous Linear Systems
13.4 Linear Autonomous Systems
13.5 Nonlinear Autonomous Systems

14 Infinite Series
14.1 Sequences and Their Limits
14.2 Infinite Series and Convergence
14.3 Tests for Convergence
14.4 Power Series
14.5 Taylor and Maclaurin Series
14.6 Series Solutions of Differential Equations

15 Fourier Series
15.1 Periodic Functions
15.2 Fourier Series
15.3 Fourier Sine and Cosine Series
15.4 Differentiation and Integration

16 Transforms
16.1 Laplace Transforms
16.2 Fourier Integrals and Fourier Transforms
16.3 Z Transforms

of Fourier Series

17 Partial Differential Equations
17.1 Partial Differential Equations and Classification
17.2 Vibrating String and the Wave Equation
17.3 Heat Equation
17.4 Vibrating Membranes
17.5 Fourier Transform Method
17.6 Laplace Transform Method

18 Numerical Analysis
18.1 Newton's Method
18.2 Numerical Integration
18.3 Curve Fitting
18.4 Solution of Differential Equations
18.5 Solution of Partial Differential Equations

19 Linear Optimization
19.1 Optimization
19.2 Simplex Method

20 Logic, Combinatorics, and Statistics
20.1 Logic and Truth Tables
20.2 Combinatorics
20.3 Data Analysis
20.4 Probability and Probability Distributions
20.5 The Central Limit Theorem
20.6 Estimation
20.7 Tests of Statistical Hypotheses
20.8 Regression and Correlation
20.9 Analysis of Variance

21 Conformal Mappings
21.1 Conformal Mappings

22 Graphs and Trees
22.1 Graphs and Digraphs
22.2 Trees and Networks

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