

Maple in Differential Equations and Boundary Value Problems

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Maple Worksheets Supplementing Edwards and Penney

Differential Equations and Boundary Value Problems - Computing and Modeling

▼ Preface

These worksheets provide a comprehensive Maple supplement to the textbook **Differential Equations and Boundary Value Problems - Computing and Modeling, Third Edition** by *C. Henry Edwards and David E. Penney*. The structure is almost identical to the textbook, and the worksheets are designed to be used in conjunction with the book itself. These Maple worksheets are intended to show how the computer algebra system Maple can be integrated and exploited as a powerful learning tool throughout a calculus course. All examples and problems presented here are very nearly reproductions of those in the textbook. They show how the use of Maple reduces the amount of time spent in routine algebraic manipulation, and thereby permits concentration instead on fundamental concepts and techniques. Moreover, these same examples are presented in these worksheets in a dynamic and interactive form. Students and instructors using Maple can change input data and conditions in an example to investigate the resulting changes in step-by-step solutions and accompanying graphs and figures. Maple thereby opens the door to all sorts of exploration and "what if" investigations that are the staple of real mathematics, but difficult to capture on a static printed page.

You are not assumed to be an experienced Maple user. Because these worksheets consist mainly of worked solutions to examples from the textbook, the textbook exposition and the Maple exposition mutually support one another, reinforcing assimilation of Maple in a relatively short period of time. Indeed, you can work through the Maple worksheets presented here simply by opening them and using the Enter key to execute in sequence the Maple commands that are already provided. But if you examine these commands, you should see how input parameters and functions given in the textbook examples have been inserted and might be altered in order to see how the results would change. My own experience is that the close resemblance of Maple syntax to standard mathematical thought and terminology shortens significantly the time required to learn Maple syntax.

To profit the most from these worksheets, the reader should work through the examples and problems as they occur and in conjunction with the textbook. You should be aware that Maple

can be used in several ways:

- Only as a calculating engine - like a powerful calculator. This means that you enter a mathematical expression which Maple subsequently executes, and you are freed from tedious computations, especially if you are not interested in where Maple gets its response to a given command.
- To present a method as a sequence of steps in much the same way as a textbook would, and to illustrate the method with simple examples. The primary purpose here is to present intermediate steps because we want to show how to solve the problem. This approach is probably relevant to most of a student's individual study and work on calculus problems.
- Unlike the study of a printed textbook, you are free to attempt larger and more realistic examples, and you can explore interactively how a solution of a problem depends on various parameters of the problem under study. The associated figures are replotted dynamically, so the graphics capabilities of Maple are extremely helpful for visualizing the behavior of the system under investigation.

In the worked examples and problems in these Maple supplements to the textbook, I have tried to illustrate these different ways to use Maple. The **Table of Contents** and all commands are [hyperlinked](#) to provide easy access to all chapters, and to the help pages for the commands used in the worksheets. Hence you can check your correct use of the commands, and can see Maple's own examples in the help pages.

These sample worksheets are intended to serve as guides when you prepare your own worksheet solutions to exercises and problems in calculus or other homework where Maple is used, and also to stimulate your personal exploration of the many facets of mathematics. But please bear this in mind: **You** must do the thinking. Maple alone only performs the calculations you direct it to carry out. It cannot think for you.

One of the beautiful features of Maple is that much can be done with few commands. Most of the examples in these worksheets involve no formal programming with Maple. You need only select and execute in turn the Maple commands that are already provided. But these commands are augmented by two additional Maple packages, not parts of the Maple software, the [calc](#) package and the [calcplot](#) package that are included here.

My hope is that these worksheets will give students easier access and additional help to make effective use of Maple in classes, labs, homework and projects, and later on in future professional positions. Maple allows the level of mathematical typesetting and the necessary word processing facilities needed for writing both homework or project assignments and professional reports.

Downloading CalculusDE with Maple

Please look at www.hpleym.no/calculus/WorksheetDE.html

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www.hpleym.no

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