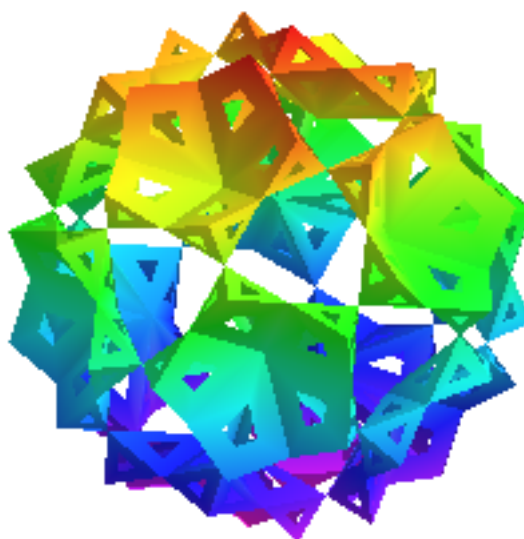




# Mathematics with Maple

## A Comprehensive E-Book

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### ▼ Preface

The main objective of these Maple worksheets, organized for use with all Maple versions from Maple 14, is to show how the computer algebra system Maple can be integrated as a valuable and powerful tool into the learning process of the material found in most mathematics textbooks at university levels. This interactive e-book can be used as a supplement to a standard textbook or on its own. All chapters in the worksheets summarize mathematical concepts and principles associated with each topic.

There are more than 600 worked examples, presented in a dynamic and interactive form. These show how Maple can be used to solve problems and also reduce the amount of time spent in routine algebraic manipulation. With Maple there is a great opportunity for exploration, clarify concepts, trying different functions, zoom in on an area, visualize mathematical objects in two or

three dimensions, perform animations etc.


I do assume that you have a basic familiarity with Maple, that you can start Maple, know how to enter commands, etc. If not, I recommend you to go through the [Getting Started Guide](#). This manual describes how to use the Standard Maple interface, which have two modes, document mode and worksheet mode. All worksheets in **Mathematics with Maple** are in worksheet mode, and all entering inputs are in text mode (maple input). Right-click the mouse and select **Convert To 2-D Math** if you want an input as you will find in a mathematics textbooks. If you need an overview of all Maple features and detailed help related to performing computations, creating plots, interactive documents etc., please consult the [User Manual](#). You can also go to [The Maple Portal](#).

My own experience is that Maple's closeness to mathematical thinking, shortens the learning time of the syntax significantly. The body of text presents only the formal mathematics concepts needed for the worked solutions of the examples, which are familiar problems found in most mathematical textbooks, so that both the commands and the style of interaction with Maple can be mastered by students and others in a relatively short period of time.

To get the most from these worksheets, you should work through the examples as they occur and in conjunction with the textbook you are using. Be aware that Maple can be used in several ways:

- Only as a solving engine. That means, you enter a mathematical expression which Maple subsequently execute, and you are freed from tedious computations, especially if you are not interested in how Maple gets the answer to a given command
- To present a method as a sequence of steps in much the same way as a textbook would, and illustrate the method with simple examples. The primary purpose here is to present intermediate steps to show you how to solve the problem. This is probably relevant in most of a student's homework solutions
- Unlike a math textbook you are free to execute larger and more realistic examples, and you can explore interactively how a solution of a problem depends on various parameters of the problem under study. The graphics capabilities are extremely helpful for visualizing the behavior of the system under investigation

In the examples in the worksheets, I have tried to illustrate these methods of using Maple. The list of contents and all commands are [hyperlinked](#) to provide easy access to all chapters, and to the help pages for the commands used in the worksheets. Then you can assure that you are using the commands correctly, see Maple's own examples on the help pages, and get further help for the commands, Maple features, packages and more. You will also find hyperlinks to famous mathematicians on the web.

A typical working session will begin by opening the contents section below. Select the link to the chapter and section you want to study. Open and press the **Enter** key repeatedly to execute in sequence the Maple commands that are already provided, or press the  key to execute the entire worksheet.

Hopefully these samples worksheets will serve as a guide when you prepare your own worksheets solutions to exercises and problems in math or other homework where Maple is used, and also inspire you to concentrate on exploring the many facets of mathematics. But, **please bear in mind: You must do the thinking, Maple alone only does the calculations and move you more quickly into applications of problem solving.**

One of the beautiful qualities of Maple is that much can be done with few commands. Most of the examples in these worksheets involve no formal programming with Maple. I have included two

extra library packages, the [calc](#) package and the [calcplot](#) package. These are not part of the Maple software. The procedures in these packages are designed for visualizing concepts from calculus and advanced calculus, without any need for you to learn the subtleties of Maple's base commands in order to use the procedures. Many of the visualization procedures go far beyond Maple's built in visualization tools.

My hope is that these worksheets will give you as a student easier access and additional help to make effective use of Maple, in classes, labs, homework and projects, and in future jobs. Maple allows the level of mathematical typesetting and also necessary word processing facilities needed for writing sets of exercises, homework and professional reports.

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